

# Mark Scheme (Results)

## January 2009

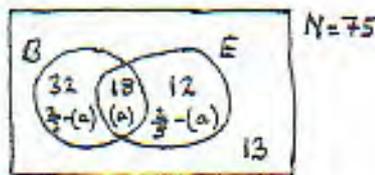
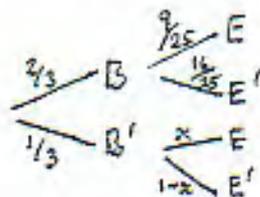
GCE

GCE Mathematics (6683/01)

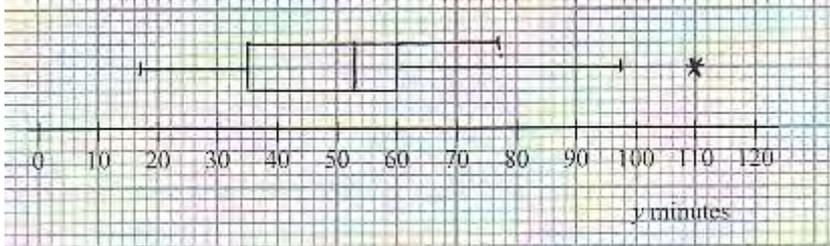
**January 2009  
6683 Statistics S1  
Mark Scheme**

Question Number	Scheme	Marks
<b>1</b>	$S_{xx} = 57.22 - \frac{(21.4)^2}{10} = 11.424$	M1 A1
(a)	$S_{xy} = 313.7 - \frac{21.4 \times 96}{10} = 108.26$	A1 (3)
(b)	$b = \frac{S_{xy}}{S_{xx}} = 9.4765\dots$ $a = \bar{y} - b\bar{x} = 9.6 - 2.14b = (-10.679\dots)$	M1 A1 M1 A1 (4)
(c)	$y = -10.7 + 9.48x$ Every (extra) <u>hour</u> spent using the programme produces about <u>9.5 marks improvement</u>	B1ft (1)
(d)	$y = -10.7 + 9.48 \times 3.3, = 20.6$	awrt 21 M1,A1 (2)
(e)	Model may not be valid since [8h is] outside the range [0.5 - 4].	B1 (1)
<b>[11]</b>		
(a)	M1 for a correct expression 1 <sup>st</sup> A1 for AWRT 11.4 for $S_{xx}$ 2 <sup>nd</sup> A1 for AWRT 108 for $S_{xy}$	
Correct answers only: One value correct scores M1 and appropriate A1, both correct M1A1A1		
(b)	1 <sup>st</sup> M1 for using their values in correct formula 1 <sup>st</sup> A1 for AWRT 9.5 2 <sup>nd</sup> M1 for correct method for $a$ (minus sign required) 2 <sup>nd</sup> A1 for equation with $a$ and $b$ AWRT 3 sf (e.g. $y = -10.68 + 9.48x$ is fine) Must have a full equation with $a$ and $b$ correct to awrt 3 sf	
(c)	B1ft for comment conveying the idea of <u><math>b</math> marks per hour</u> . Must mention value of $b$ but can fit their value of $b$ . No need to mention “extra” but must mention “marks” and “hour(s)” e.g. “...9.5 times per hour...” scores B0	
(d)	M1 for sub $x = 3.3$ into their regression equation from the end of part (b) A1 for awrt 21	
(e)	B1 for a statement that says or implies that it may <u>not</u> be valid because <u>outside the range</u> . They do not have to mention the values concerned here namely 8 h or 0.5 - 4	

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<p>2</p> <p>(a)</p> <p>(b)</p> <p>(c)</p>	<p><math>E = \text{take regular exercise}</math>      <math>B = \text{always eat breakfast}</math></p> <p><math>P(E \cap B) = P(E   B) \times P(B)</math></p> <p><math>= \frac{9}{25} \times \frac{2}{3} = 0.24</math>    or    <math>\frac{6}{25}</math>    or    <math>\frac{18}{75}</math></p> <p><math>P(E \cup B) = \frac{2}{3} + \frac{2}{5} - \frac{6}{25}</math>      or <math>P(E'   B')</math>      or <math>P(B' \cap E)</math>      or <math>P(B \cap E')</math></p> <p><math>= \frac{62}{75}</math>         <math>= \frac{13}{25}</math>         <math>= \frac{12}{75}</math>         <math>= \frac{32}{75}</math></p> <p><math>P(E' \cap B') = 1 - P(E \cup B) = \frac{13}{75}</math>    or    <math>0.17\bar{3}</math></p> <p><math>P(E   B) = 0.36 \neq 0.40 = P(E)</math>    or    <math>P(E \cap B) = \frac{6}{25} \neq \frac{2}{5} \times \frac{2}{3} = P(E) \times P(B)</math></p> <p>So <math>E</math> and <math>B</math> are <u>not</u> statistically independent</p>	<p>M1</p> <p>A1    (2)</p> <p>M1</p> <p>A1</p> <p>M1 A1    (4)</p> <p>M1</p> <p>A1    (2)</p> <p>[8]</p>
<p>(a)</p> <p>(b)</p> <p>(c)</p>	<p>M1    for <math>\frac{9}{25} \times \frac{2}{3}</math> or <math>P(E B) \times P(B)</math> <u>and</u> at least one correct value seen. A1 for 0.24 or exact equiv. NB <math>\frac{2}{5} \times \frac{2}{3}</math> alone or <math>\frac{2}{5} \times \frac{9}{25}</math> alone scores M0A0. Correct answer scores full marks.</p> <p>1<sup>st</sup> M1    for use of the addition rule. Must have 3 terms and some values, can ft their (a) <u>Or</u> a full method for <math>P(E'   B')</math> requires <math>1 - P(E B')</math> and equation for <math>P(E B')</math>: <math>(a) + \frac{x}{3} = \frac{2}{5}</math> <u>Or</u> a full method for <math>P(B' \cap E)</math> <u>or</u> <math>P(B \cap E')</math> [ or other valid method]</p> <p>2<sup>nd</sup> M1    for a method leading to answer e.g. <math>1 - P(E \cup B)</math> <u>or</u> <math>P(B') \times P(E'   B')</math> <u>or</u> <math>P(B') - P(B' \cap E)</math> <u>or</u> <math>P(E') - P(B \cap E')</math></p> <p><u>Venn Diagram</u> 1<sup>st</sup> M1 for diagram with attempt at <math>\frac{2}{5} - P(B \cap E)</math> or <math>\frac{2}{3} - P(B \cap E)</math>. Can ft their (a)</p> <p>1<sup>st</sup> A1    for a correct first probability as listed or 32, 18 and 12 on Venn Diagram</p> <p>2<sup>nd</sup> M1    for attempting 75 - their (18 + 32 + 12)</p> <p>M1    for identifying suitable values to test for independence e.g. <math>P(E) = 0.40</math> and <math>P(E B) = 0.36</math> <u>Or</u> <math>P(E) \times P(B) = \dots</math> and <math>P(E \cap B) = \text{their (a)}</math> [but their (a) <math>\neq \frac{2}{5} \times \frac{2}{3}</math>]. Values seen somewhere</p> <p>A1    for correct values and a correct comment</p> <p><b>Diagrams</b> You may see these or find these useful for identifying probabilities.</p>	<p><b>Common Errors</b></p> <p>(a) <math>\frac{9}{25}</math> is M0A0</p> <p>(b) <math>P(E \cup B) = \frac{53}{75}</math> scores M1A0</p> <p><math>1 - P(E \cup B) = \frac{22}{75}</math> scores M1A0</p> <p>(b) <math>P(B') \times P(E') = \frac{1}{3} \times \frac{3}{5}</math> scores 0/4</p>



Question Number	Scheme	Marks																		
3	<p>(a) <math>E(X) = 0 \times 0.4 + 1 \times 0.3 + \dots + 3 \times 0.1, = 1</math></p> <p>(b) <math>F(1.5) = [P(X \leq 1.5) =] P(X \leq 1), = 0.4 + 0.3 = 0.7</math></p> <p>(c) <math>E(X^2) = 0^2 \times 0.4 + 1^2 \times 0.3 + \dots + 3^2 \times 0.1, = 2</math>  <math>\text{Var}(X) = 2 - 1^2, = 1</math> (*)</p> <p>(d) <math>\text{Var}(5 - 3X) = (-3)^2 \text{Var}(X), = 9</math></p> <p>(e)</p> <table border="1" style="margin-left: auto; margin-right: auto;"> <thead> <tr> <th>Total</th> <th>Cases</th> <th>Probability</th> </tr> </thead> <tbody> <tr> <td rowspan="3">4</td> <td><math>(X = 3) \cap (X = 1)</math></td> <td><math>0.1 \times 0.3 = 0.03</math></td> </tr> <tr> <td><math>(X = 1) \cap (X = 3)</math></td> <td><math>0.3 \times 0.1 = 0.03</math></td> </tr> <tr> <td><math>(X = 2) \cap (X = 2)</math></td> <td><math>0.2 \times 0.2 = 0.04</math></td> </tr> <tr> <td rowspan="2">5</td> <td><math>(X = 3) \cap (X = 2)</math></td> <td><math>0.1 \times 0.2 = 0.02</math></td> </tr> <tr> <td><math>(X = 2) \cap (X = 3)</math></td> <td><math>0.2 \times 0.1 = 0.02</math></td> </tr> <tr> <td>6</td> <td><math>(X = 3) \cap (X = 3)</math></td> <td><math>0.1 \times 0.1 = 0.01</math></td> </tr> </tbody> </table> <p>Total probability = <math>0.03 + 0.03 + 0.04 + 0.02 + 0.02 + 0.01 = 0.15</math></p>	Total	Cases	Probability	4	$(X = 3) \cap (X = 1)$	$0.1 \times 0.3 = 0.03$	$(X = 1) \cap (X = 3)$	$0.3 \times 0.1 = 0.03$	$(X = 2) \cap (X = 2)$	$0.2 \times 0.2 = 0.04$	5	$(X = 3) \cap (X = 2)$	$0.1 \times 0.2 = 0.02$	$(X = 2) \cap (X = 3)$	$0.2 \times 0.1 = 0.02$	6	$(X = 3) \cap (X = 3)$	$0.1 \times 0.1 = 0.01$	<p>M1, A1 (2)</p> <p>M1, A1 (2)</p> <p>M1, A1 M1, A1cso (4)</p> <p>M1, A1 (2)</p> <p>B1B1B1</p> <p>M1 A1</p> <p>A1 (6) [16]</p>
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6	$(X = 3) \cap (X = 3)$	$0.1 \times 0.1 = 0.01$																		
ALT	<p>(a) M1 for at least 3 terms seen. Correct answer only scores M1A1. Dividing by <math>k (\neq 1)</math> is M0.</p> <p>(b) M1 for <math>F(1.5) = P(X \leq 1)</math>. [Beware: <math>2 \times 0.2 + 3 \times 0.1 = 0.7</math> but scores M0A0]</p> <p>(c) 1<sup>st</sup> M1 for at least 2 non-zero terms seen. <math>E(X^2) = 2</math> alone is M0. Condone calling <math>E(X^2) = \text{Var}(X)</math>.                      1<sup>st</sup> A1 is for an answer of 2 or a fully correct expression.                      2<sup>nd</sup> M1 for <math>-\mu^2</math>, condone <math>2 - 1</math>, unless clearly <math>2 -</math>. Allow <math>2 - \mu^2</math>, with <math>= 1</math> even if <math>E(X) \neq 1</math>                      2<sup>nd</sup> A1 for a fully correct solution with no incorrect working seen, <b>both</b> Ms required.  <math display="block">\sum (x - \mu)^2 \times P(X = x)</math></p> <p>1<sup>st</sup> M1 for an attempt at a full list of <math>(x - \mu)^2</math> values and probabilities. 1<sup>st</sup> A1 if all correct                      2<sup>nd</sup> M1 for at least 2 non-zero terms of <math>(x - \mu)^2 \times P(X = x)</math> seen. 2<sup>nd</sup> A1 for <math>0.4 + 0.2 + 0.4 = 1</math></p> <p>(d) M1 for use of the correct formula. <math>-3^2 \text{Var}(X)</math> is M0 unless the final answer is <math>&gt;0</math>.</p> <p>(e) Can follow through their <math>\text{Var}(X)</math> for M1</p>																			
ALT	<p>1<sup>st</sup> B1 for all cases listed for a total of 4 or 5 or 6 . e.g. (2,2) counted twice for a total of 4 is B0                      2<sup>nd</sup> B1 for all cases listed for 2 totals                      3<sup>rd</sup> B1 for a complete list of all 6 cases } These may be highlighted in a table</p> <p><u>Using Cumulative probabilities</u>                      1<sup>st</sup> B1 for one or more cumulative probabilities used e.g. 2 then 2 or more or 3 then 1 or more                      2<sup>nd</sup> B1 for both cumulative probabilities used. 3<sup>rd</sup> B1 for a complete list 1, 3; 2, <math>\geq 2</math>; 3, <math>\geq 1</math>                      M1 for one correct pair of correct probabilities multiplied                      1<sup>st</sup> A1 for all 6 correct probabilities listed (0.03, 0.03, 0.04, 0.02, 0.02, 0.01) needn't be added.                      2<sup>nd</sup> A1 for 0.15 or exact equivalent only as the final answer.</p>																			

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4	<p>(a) <math>Q_2 = 53, Q_1 = 35, Q_3 = 60</math></p> <p>(b) <math>Q_3 - Q_1 = 25 \Rightarrow Q_1 - 1.5 \times 25 = -2.5</math> (no outlier)  <math>Q_3 + 1.5 \times 25 = 97.5</math> (so 110 is an outlier)</p> <p>(c) </p> <p>(d) <math>\sum y = 461, \sum y^2 = 24\ 219 \therefore S_{yy} = 24219 - \frac{461^2}{10}, = 2966.9</math> (*)</p> <p>(e) <math>r = \frac{-18.3}{\sqrt{3463.6 \times 2966.9}}</math> or <math>\frac{-18.3}{3205.64\dots} = -0.0057</math> AWR T - 0.006 or <math>-6 \times 10^{-3}</math></p> <p>(f) <math>r</math> suggests correlation is close to zero so parent's claim is not justified</p>	<p>B1, B1, B1 (3)</p> <p>M1</p> <p>A1 (2)</p> <p>M1</p> <p>A1ft</p> <p>A1ft (3)</p> <p>B1, B1, B1cso (3)</p> <p>M1 A1 (2)</p> <p>B1 (1)</p> <p>[14]</p>
	<p>(a) 1<sup>st</sup> B1 for median                  2<sup>nd</sup> B1 for lower quartile                  3<sup>rd</sup> B1 for upper quartile</p> <p>(b) M1 for attempt to find one limit                  A1 for both limits found and correct. No explicit comment about outliers needed.</p> <p>(c) M1 for a box and two whiskers                  1<sup>st</sup> A1ft for correct position of box, median and quartiles. Follow through their values.                  2<sup>nd</sup> A1ft for 17 and 77 or "their" 97.5 and *. If 110 is not an outlier then score A0 here.                  Penalise no gap between end of whisker and outlier. Must label outlier, needn't be with *.  <b>Accuracy</b> should be within the correct square so 97 or 98 will do for 97.5</p> <p>(d) 1<sup>st</sup> B1 for <math>\sum y</math> N.B. <math>(\sum y)^2 = 212521</math> and can imply this mark                  2<sup>nd</sup> B1 for <math>\sum y^2</math> or at least three correct terms of <math>\sum (y - \bar{y})^2</math> seen.                  3<sup>rd</sup> B1 for complete correct expression seen leading to 2966.9. So all 10 terms of <math>\sum (y - \bar{y})^2</math></p> <p>(e) M1 for attempt at correct expression for <math>r</math>. Can ft their <math>S_{yy}</math> for M1.</p> <p>(f) B1 for comment <u>rejecting</u> parent's claim on basis of <u>weak or zero</u> correlation                  Typical error is "negative correlation so comment is true" which scores B0                  Weak negative or weak positive correlation is OK as the basis for their rejection.</p>	

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5	<p>(a) 8-10 hours: width = 10.5 - 7.5 = 3 represented by 1.5cm                      16-25 hours: width = 25.5 - 15.5 = 10 so represented by <u>5 cm</u>                      8- 10 hours: height = fd = 18/3 = 6 represented by 3 cm                      16-25 hours: height = fd = 15/10 = 1.5 represented by <u>0.75 cm</u></p> <p>(b) <math>Q_2 = 7.5 + \frac{(52-36)}{18} \times 3 = 10.2</math>  <math>Q_1 = 5.5 + \frac{(26-20)}{16} \times 2 [= 6.25 \text{ or } 6.3]</math> or <math>5.5 + \frac{(26.25-20)}{16} \times 2 [= 6.3]</math>  <math>Q_3 = 10.5 + \frac{(78-54)}{25} \times 5 [= 15.3]</math> or <math>10.5 + \frac{(78.75-54)}{25} \times 5 [= 15.45 \text{ \15.5}]</math>                      IQR = (15.3 - 6.3) = 9</p> <p>(c) <math>\sum fx = 1333.5 \Rightarrow \bar{x} = \frac{1333.5}{104} =</math> AWRT <u>12.8</u></p> <p>(d) <math>\sum fx^2 = 27254 \Rightarrow \sigma_x = \sqrt{\frac{27254}{104} - \bar{x}^2} = \sqrt{262.05 - \bar{x}^2}</math> AWRT <u>9.88</u>  <math>Q_3 - Q_2 [= 5.1] &gt; Q_2 - Q_1 [= 3.9]</math> or <math>Q_2 &lt; \bar{x}</math></p> <p>(e) So data is positively skew</p> <p>Use median and IQR,                      since data is skewed <u>or</u> not affected by extreme values or outliers</p>	<p>B1 M1 A1 (3)</p> <p>M1 A1</p> <p>A1</p> <p>A1 A1ft (5)</p> <p>M1 A1</p> <p>M1 A1 (4)</p> <p>B1ft dB1 (2)</p> <p>B1 B1 (2)</p> <p>[16]</p>
	<p>(a) M1 For attempting both frequency densities <math>\frac{18}{3} (= 6)</math> and <math>\frac{15}{10}</math>, <u>and</u> <math>\frac{15}{10} \times \text{SF}</math>, where <math>\text{SF} \neq 1</math></p> <p>(b) NB Wrong class widths( 2 and 9) gives <math>\frac{h}{1.66\dots} = \frac{3}{9} \rightarrow h = \frac{5}{9}</math> or 0.55... and scores M1A0</p> <p>M1 for identifying correct interval and a correct fraction e.g. <math>\frac{\frac{1}{2}(104)-36}{18}</math>. Condone 52.5 or 53</p> <p>1<sup>st</sup> A1 for 10.2 for median. Using (n + 1) allow awrt 10.3</p> <p>2<sup>nd</sup> A1 for a correct expression for either <math>Q_1</math> or <math>Q_3</math> (allow 26.25 and 78.75) NB: <u>Must see</u></p> <p>3<sup>rd</sup> A1 for correct expressions for both <math>Q_1</math> and <math>Q_3</math> <u>some</u></p> <p>(c) 4<sup>th</sup> A1ft for IQR, ft their quartiles. Using (n + 1) gives 6.28 and 15.45 <u>method</u></p> <p>1<sup>st</sup> M1 for attempting <math>\sum fx</math> and <math>\bar{x}</math></p> <p>(d) 2<sup>nd</sup> M1 for attempting <math>\sum fx^2</math> and <math>\sigma_x, \sqrt{\quad}</math> is needed for M1. Allow s = awrt 9.93</p> <p>1<sup>st</sup> B1ft for suitable test, values need not be seen but statement must be compatible with values used. Follow through their values</p> <p>2<sup>nd</sup> dB1 Dependent upon their test showing positive and for stating positive skew                      If their test shows negative skew they can score 1<sup>st</sup> B1 but lose the second</p> <p>(e) 1<sup>st</sup> B1 for choosing median and IQR. Must mention <u>both</u>. } <u>Award independently</u>                      2<sup>nd</sup> B1 for suitable reason }                      e.g. “use median because data is skewed” scores B0B1 since IQR is not mentioned</p>	

Question Number	Scheme	Marks
<p>6 (a)</p> <p>(b)</p> <p>(c)</p> <p>(d)</p>	$P(X < 39) = P\left(Z < \frac{39-30}{5}\right)$ $= P(Z < 1.8) = \underline{0.9641} \quad (\text{allow awrt } 0.964)$ $P(X < d) = P\left(Z < \frac{d-30}{5}\right) = 0.1151$ $1 - 0.1151 = 0.8849 \quad (\text{allow } \pm 1.2)$ $\Rightarrow z = -1.2$ $\therefore \frac{d-30}{5} = -1.2 \quad \underline{d = 24}$ $P(X > e) = 0.1151 \quad \text{so } e = \mu + (\mu - \text{their } d) \text{ or } \frac{e-30}{5} = 1.2 \text{ or } - \text{their } z$ $\underline{e = 36}$ $P(d < X < e) = 1 - 2 \times 0.1151$ $= 0.7698 \quad \text{AWRT } \underline{0.770}$	<p>M1</p> <p>A1 (2)</p> <p>M1</p> <p>B1</p> <p>M1A1 (4)</p> <p>M1</p> <p>A1 (2)</p> <p>M1</p> <p>A1 (2)</p> <p>[10]</p>
	<p><b>Answer only scores all marks in each section BUT check (b) and (c) are in correct order</b></p> <p>(a) M1 for standardising with <math>\sigma</math>, <math>z = \pm \frac{39-30}{5}</math> is OK  A1 for 0.9641 or awrt 0.964 but if they go on to calculate <math>1 - 0.9641</math> they get M1A0</p> <p>(b) 1<sup>st</sup> M1 for attempting <math>1 - 0.1151</math>. Must be seen in (b) in connection with finding <math>d</math>  B1 for <math>z = \pm 1.2</math>. They must state <math>z = \pm 1.2</math> or imply it is a <math>z</math> value by its use.  This mark is only available in part (b).  2<sup>nd</sup> M1 for <math>\left(\frac{d-30}{5}\right) =</math> their negative <math>z</math> value (or equivalent)</p> <p>(c) M1 for a full method to find <math>e</math>. If they used <math>z = 1.2</math> in (b) they can get M1 for <math>z = \pm 1.2</math> here  If they use symmetry about the mean <math>\mu + (\mu - \text{their } d)</math> then ft their <math>d</math> for M1  Must explicitly <u>see</u> the method used unless the answer is correct.</p> <p>(d) M1 for a complete method or use of a correct expression e.g. “their 0.8849” - 0.1151  <u>or If their <math>d &lt;</math> their <math>e</math> using their values with <math>P(X &lt; e) - P(X &lt; d)</math></u>  If their <math>d \geq</math> their <math>e</math> then they can only score from an argument like <math>1 - 2 \times 0.1151</math>  A negative probability or probability <math>&gt; 1</math> for part (d) scores M0A0</p>	